DESIGN CHARTS FOR STABILITY ANALYSIS OF SLOPES REINFORCED WITH EXTENSIBLE REINFORCEMENT BASED ON A CIRCULAR SLIP SURFACE

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ABSTRACT

This paper examines the problem of designing steepened slopes which are reinforced with extensible reinforcing elements such as geosynthetics. A variety of analytical methods are presently in use for designing such slopes, each relying on some simplifying assumptions to render the moderately complex statics problem solvable. This paper presents a solution which assumes a circular failure surface and incorporates the tensile contribution of the reinforcing elements tangential to the failure surface. A simple computer code was developed to obtain the critical slip circle geometry and required tensile resistance of the reinforcing elements for a given factor of safety against rotational instability. The results are presented in chart form, and a worked example is presented to illustrate the use of the charts.

INTRODUCTION

The problem of the reinforced steepened slope is moderately complex, and defies an exact solution by standard analytical methods. Proprietary design charts are available with approximate solutions for specific reinforcing products. This paper presents general design charts for simple slopes reinforced with any extensible reinforcing elements.

Slopes can be reinforced by a wide variety of methods. Reinforcing elements installed within the slope itself can be broadly divided into two types for the purposes of this paper: rigid elements, such as steel rods or mesh, and extensible elements, such as geosynthetics. Extensible reinforcing elements are considered here.

This paper does not address the problems of external stability, which are easily treated with standard analytical methods, and does not explicitly address the problem of pullout resistance, which can be considered in a number of approximate ways. Internal stability against tensile failure of the extensible reinforcing elements is analysed with the simplifying assumption that a circular slip surface is observed at failure.

This paper makes no attempt to prove that a circular failure surface is correct, nor does it suggest that the assumption is intrinsically correct; rather, it is suggested that by conducting a thorough search of all plausible circular failure surfaces, the "right" answer is approximated, thus providing an answer suitable for engineering design.

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GENERAL DESIGN PHILOSOPHY

The reinforced slope problem requires consideration of a wide variety of factors, some of which include: foundation soil properties, slope geometry, backfill soil properties, drainage, reinforcement strength and stress-strain characteristics, soil-reinforcement interface behaviour, service loads, and required safety factors or limit states. Figure 1 illustrates some of the information required for design.

The designer must check the reinforced slope against a variety of failure mechanisms, which can be grouped broadly into internal stability and external stability modes. External stability checks include sliding, global instability (deep seated failure), external seismic loading and settlement. Internal stability checks are conducted to determine the stability against tensile failure and pullout of the reinforcement under dead, live and seismic loading conditions.

The exact methodology used to design reinforced slopes varies with the prevailing design philosophy. European standards (British Standards Institution, 1991, Schardin-Liedtke, 1990, Studer and Meier, 1986) favour a limit states approach, in which partial safety factors are applied to the material properties and loads, and the slope is expected to perform within prescribed ultimate and serviceability limit states. North American approaches appear to favour a mix of some partial factors and overall safety factors more normally associated with common geotechnical design practice. The US Department of Transportation (National Highway Institute, 1989) provides specific safety factors to be applied to sliding, overall stability, seismic loading and internal stability of the reinforced slope. They also provide material factors to be used for the allowable geosynthetic strength and pullout resistance. The Canadian Foundation Engineering Manual (Canadian Geotechnical Society, 1990) provides limited design guidance for reinforced slopes. The designer is required to check external and internal stability, but no methods are specified, and no safety factors are provided for the material properties or the various failure mechanisms.

Several methods have been used to calculate internal stability, most of which are based on the simplifying assumptions of limit equilibrium analysis, in which the slope is assumed to behave as a rigid-plastic mass. The limit equilibrium methods can be grouped into two major categories: lateral earth pressure methods (ie. John, 1987, and Resl, 1990) and slope stability methods (several authors). The latter method is specified in European and North American standards. It is also possible to analyse the reinforced slope with finite elements, or other such sophisticated analytical methods; however, this approach is considered too tedious and costly for all but the most important structures.

Slope stability methods of limit equilibrium analysis are based on the concept of comparing mobilized resistance to driving forces. This analysis can be conducted assuming a variety of failure modes, including single wedge, two part wedge, circle or log-spiral. The actual failure surface is not likely to be any of these shapes exactly, but rather more likely depends on the spacing and relative strength and stiffness of the reinforcing elements in comparison with the strength and stiffness of the surrounding soil.

The factor of safety against rotational instability is calculated as the ratio of resisting moment, which includes soil shear resistance and geosynthetic tensile contribution, to driving moment, which is derived from the weight of the structure plus any applied loads. The structure is designed to exceed some specific factor of safety.

Some standards allow the tensile contribution of the extensible reinforcement to be assumed to act tangential to the failure surface, as the geosynthetic is assumed to have elongated and realigned along the potential failure surface at the point of incipient failure. This concept is illustrated in Figure 2. It should be noted that if this assumption is made, it becomes important to re-examine the selection of soil shear strength properties, as the strain required to realign the geosynthetic may be high enough that the peak shear strength does not apply. In this paper, it is suggested that a constant volume friction angle be used in the absence of field data or test data supporting the use of a higher peak value.

If a circular failure surface is assumed in conjunction with the realignment of the geosynthetic, the analysis is greatly simplified, as the entire tensile contribution due to the reinforcement can be included as a single resisting moment equal to the total tensile resistance multiplied by the radius of the slip circle.

METHOD OF ANALYSIS

The internal stability results presented in this paper were calculated using a simple computer program based on a slope stability limit equilibrium approach with circular slip surfaces, assuming the realignment of geosynthetic tensile resistance tangential to the slip surface. The program was used to estimate the tensile force required to provide a given reinforced factor of safety against rotational failure, using Bishop's Simplified Method of Slices, and incorporating the additional resisting contribution due to the reinforcement.

The factor of safety against rotational failure for the reinforced slope can be written in simplified form as:

(1)

$$FS_R = \frac{M_R}{M_D} = \frac{(T + \Sigma\tau_i b_i)R}{\Sigma W_i l_i}$$

where:
FS_R = reinforced factor of safety against rotational failure
M_D = driving moment
M_R = resisting moment
T = total tensile contribution of the geosynthetic reinforcing elements = ΣT_n
T_n = tensile contribution of the "n"th layer of reinforcement
τ_i = shear strength of the soil along the failure surface at the base of the "i"th slice
b_i = length along the slip surface at the base of the "i"th slice
R = radius of the critical slip circle
W_i = weight of the "i"th slice
l_i = moment arm from the centre of the circle to the centre of mass of the "I"th slice

Figure 3 illustrates some of these variables. Equation (1) can be rearranged to provide the required tensile resistance for a given factor of safety against rotational failure as:

$$T = \frac{(FS_R - FS_U)M_D}{R} \tag{2}$$

where:
FS_U = $(R\Sigma\tau_i b_i) / (\Sigma W_i l_i)$
= the factor of safety of the unreinforced slope corresponding to the failure surface being examined for the reinforced slope. Note that this does not generally correspond to the unreinforced slope's actual factor of safety, as determined for the critical slip surface for the unreinforced slope.

A computer program was written to perform calculations for reinforced slopes as described above. This program calculates the tensile resistance, T, required to provide the desired factor of safety, FS_R, for a large number of potential failure surfaces. A methodical search is conducted to determine the circle resulting in the largest required tensile resistance for a given factor of safety.

The maximum tensile resistance is estimated by checking a large number of potential slip circles. Each circle originates from the toe of the slope, and emerges some distance, X_T, from the crest. A line drawn from the toe to the point where the circle emerges behind the crest makes an angle, ψ, with the horizontal, as illustrated in Figure 4. Since the circle must pass through the toe, the circle geometry can be fully described by the radius, R, and either X_T or ψ.

The computer program checks circles with various values of R and X_T looking for the circle which requires the largest tensile force for the given factor of safety, FS_R. Radius is varied from very short (ie. centre just outside the slope) to very long (ie. resulting in a single wedge) in discrete increments. X_T is also varied in discrete increments from zero until it is clear that the trend is for decreasing T. In this manner, the maximum required tensile resistance and corresponding slip circle geometry could be obtained for a given slope and given required factor of safety, FS_R.

It should be noted that, for a given slope and required factor of safety, the calculated tension increases with X_T to a point, after which it decreases. Varying the radius of the slip circle also has a significant effect on T. The general trends are shown in Figure 5.

The tensile resistance, T, determined as the greatest value obtained for all circles checked, is then used to determine the total amount of reinforcement required. The corresponding critical failure surface is used to determine the required length of reinforcement. Only that length of reinforcement which extends beyond the failure surface can mobilize resistance to failure. The magnitude of mobilized resistance is limited by the strength of the reinforcement or the pullout resistance of that length of reinforcement extending beyond the failure surface.

The computer program was written to analyse simple slopes conforming to the following criteria:

(1) Competent foundation soil (therefore only toe circles were examined)
(2) Cohesionless, well-drained backfill (no pore pressures)
(3) Horizontal crest
(4) No surcharge
(5) No seismic loads

The computer analysis allowed the variation of five variables: slope height, bulk weight, slope angle, friction angle and desired factor of safety. The choice of height and bulk weight are irrelevant for cohesionless soil. The geometry of the critical slip surface is independent of these factors, and the calculated total tension, T, varies linearly with bulk weight and the square of the height. For ease of calculations, the program analysed slopes 5 m high, with backfill bulk weight 20 kN/m^3. A batch program was written to evaluate slopes with the following ranges in key properties:

(1) Slope angle - 25 to 85 degrees at 5 degree increments
(2) FS_R - 1.0, 1.3, 1.5
(3) Friction angle, ϕ, - 15 to 45 degrees at 5 degree increments

GENERAL OBSERVATIONS AND RESULTS

It is interesting to examine the effect of varying slope angle, friction angle and desired factor of safety. Figure 6 shows the effect of varying the desired factor of safety, FS_R, while keeping all other variables constant. The critical circle moves away from the crest of the slope, and required tension, T, increases, as FS_R increases. The relationships between slip circle geometry, T and FS_R are not linear. Figure 7 shows the effect of changing friction angle while holding the other variables constant. As the friction angle is reduced, the critical circle moves away from the crest. The total required tension also increases. Figure 8 shows the effect of varying slope angle while holding friction angle constant. It is seen that for steep slopes, the critical circle approximates a linear wedge. As the slope angle decreases, the radius of the critical circle becomes shorter. If the slope angle is reduced enough, the radius increases again.

Complete results of the computer analysis are plotted in the charts in Figures 9 to 17. There are three charts each for FS_R = 1.0, 1.3 and 1.5. The first chart for each FS_R gives the required tension, T, for a given slope and various friction angles. Tension is determined by multiplying the chart value by the backfill bulk weight and the square of the slope height. The second chart gives the angle of exit for the critical circle, ψ, in degrees. The third chart gives the radius, R, of the critical circle. The radius is calculated by multiplying the chart value by the slope height.

A WORKED EXAMPLE

The following worked example presents one method of using the design charts in Figures 9 to 17 for the design of reinforced slopes. The reader is cautioned to make his own judgement regarding the correct use of safety factors for the given loads, soil, geosynthetic, failure condition and jurisdiction. This example considers only internal stability for the static case; it is assumed that other checks have been or will be made.

Example: design a 7.5 m high reinforced wall with a slope of 2V:1H. Available backfill material is Fraser River sand. Assume a friction angle of 33° and compacted bulk weight of 18.5 kN/m³. Design the slope for a reinforced factor of safety of 1.3 in accordance with USFHWA design guidance. Three geosynthetic materials are available, with design tensile strengths of 10, 20 and 50 kN/m, and friction coefficients with the compacted backfill of 0.9tanϕ.

Step 1: determine T from Figure 12 to be

$$\begin{aligned} T &= 0.135(7.5\text{m})^2(18.5\text{kN/m}^3) \\ &= 140.5\text{kN/m} \end{aligned}$$

Step 2: determine the angle of exit of the critical circle from Figure 13 as 44°.

Step 3: determine the radius of the critical circle from Figure 14 as:

$$\begin{aligned} R &= 2(7.5m) \\ &= 15m \end{aligned}$$

Note: for simplicity, assuming R=1.75H will always be conservative, and assuming R=2H will generally give a slightly conservative result for the majority of slopes.

Plot the critical circle superimposed on the slope to aid visualization of the problem solution, if desired.

Step 4: choose a geosynthetic and determine the correct number of layers. Note that the selection of product and spacing can be done in a variety of different ways to achieve the required result. The final choice can be made based on cost or other factors, and the designer should bear in mind the requirement for secondary reinforcement if the primary layers are spaced more than 0.6 m apart. In this case, choose 15 layers of the 10 kN/m product.

Step 5: determine layer spacing. The assumptions used to simplify the analysis (ie. tensile resistance tangential to the failure surface) allow the designer to space the reinforcement in any desired arrangement to provide the required tension, T, so long as other factors do not govern. It is generally recommended to space the reinforcement more closely near the base (Fannin, 1990, National Highway Institute, 1989). This may allow better uniformity of strain distribution in the reinforcement layers, resulting in a better performing structure. This also allows a more efficient structure in that allowable pullout resistance per unit length of reinforcement increases with depth. For this example, assume 15 layers spaced at 0.5 m starting 0.5 m below the crest. Secondary reinforcement will not be required.

Step 6: determine reinforcement length for adequate pullout resistance. Note that total length is the length embedded beyond the critical circle plus the length within the circle. If the layers are to be constructed with uniform length, the length of the upper layer will likely govern. USFHWA (National Highway Institute, 1989) requires FS>1.5 for pullout for granular soil. If the allowable pullout resistance is taken as:

$$(T_n)_{allow} = \frac{(2L_E\sigma_v)(0.9\tan\phi)}{FS}$$

where:
$(T_n)_{allow}$ = allowable pullout resistance for the "n"th layer
L_E = embedment length beyond the critical slip surface

then by rearranging this equation we can calculate the required total length of reinforcement as approximately 5.3 m.

Note that different solutions with shorter overall reinforcement length can be obtained by concentrating the reinforcement in the lower part of the slope, and treating the upper part of the slope as a lower version of the same slope (recalling that T is a function of height squared), with lower strength reinforcement and shorter reinforcement lengths.

CONCLUSION

This paper has presented a simplified approach for designing simple reinforced slopes under certain conditions. The charts may be used for the rapid evaluation of a range of design problems.

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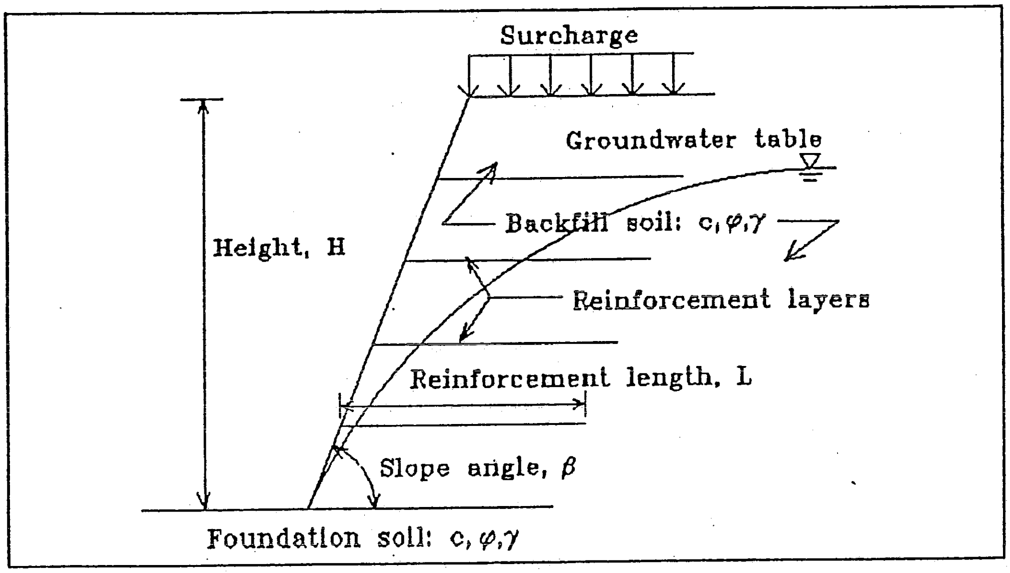

Figure 1 - Reinforced Slope Criteria

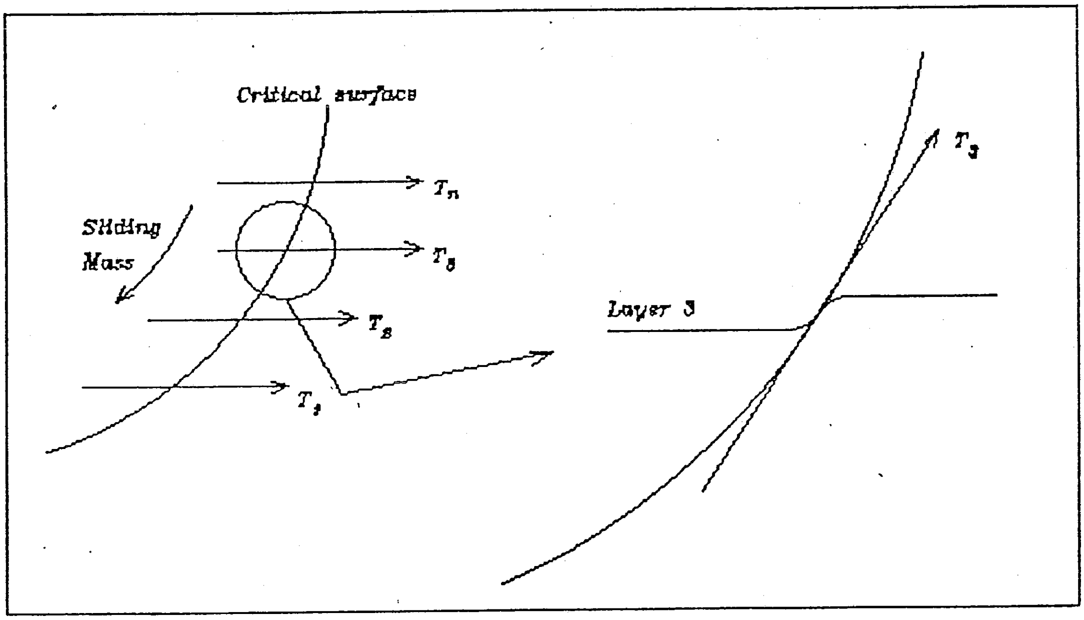

Figure 2 - Realignment of Tensile Force at Incipient Failure

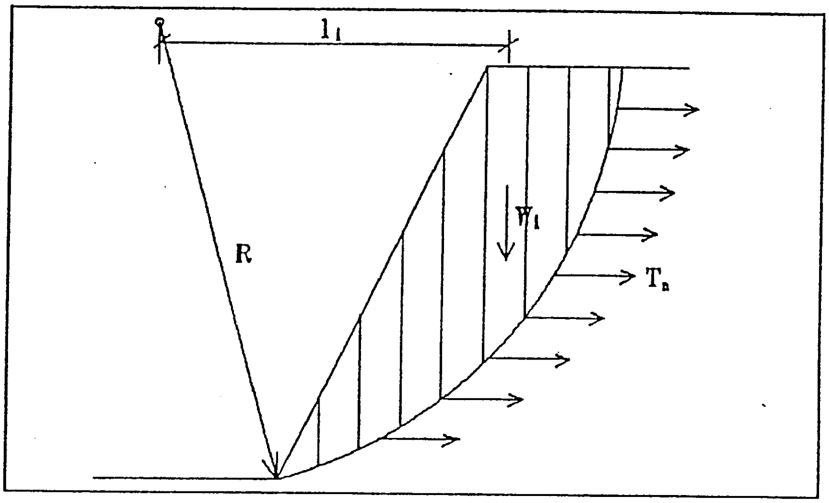

Figure 3 - Method of Slices Incorporating Tensile Resistance

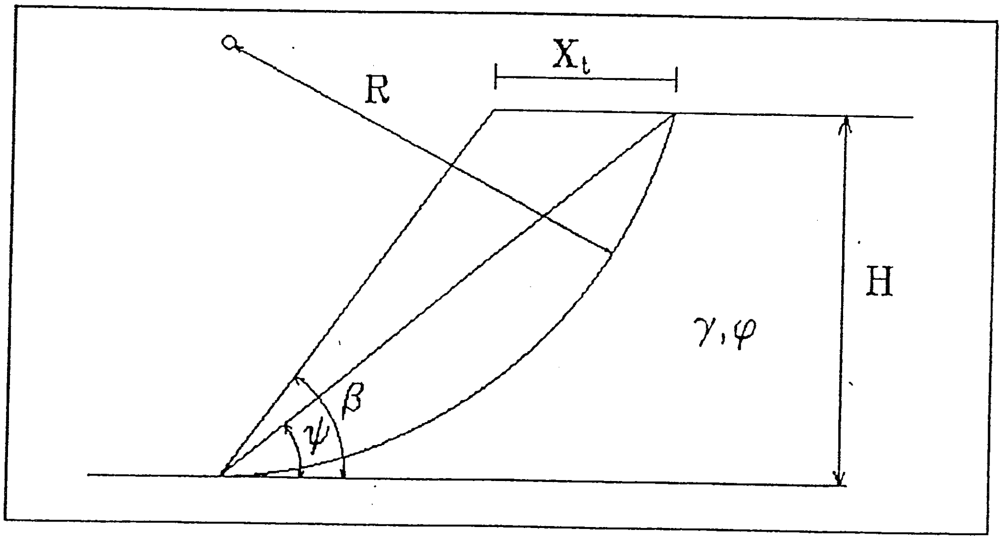

Figure 4 - Example Failure Circle

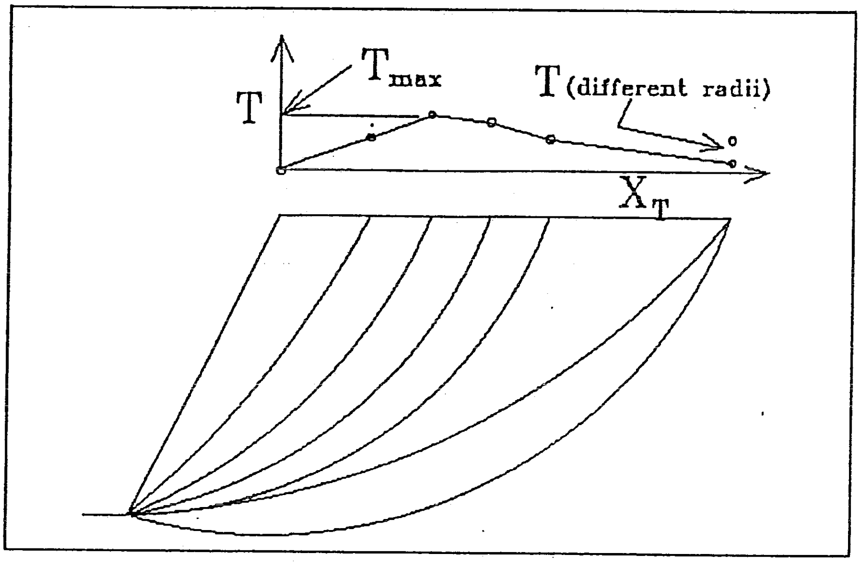

Figure 5 - Tension Required for Various Slip Circles

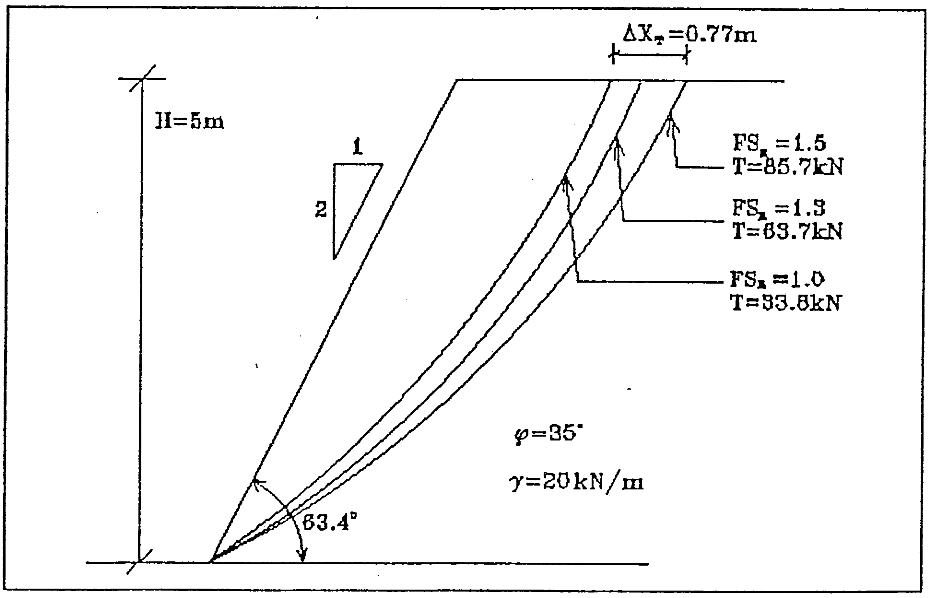

Figure 6 - Effect of Varying FS_R

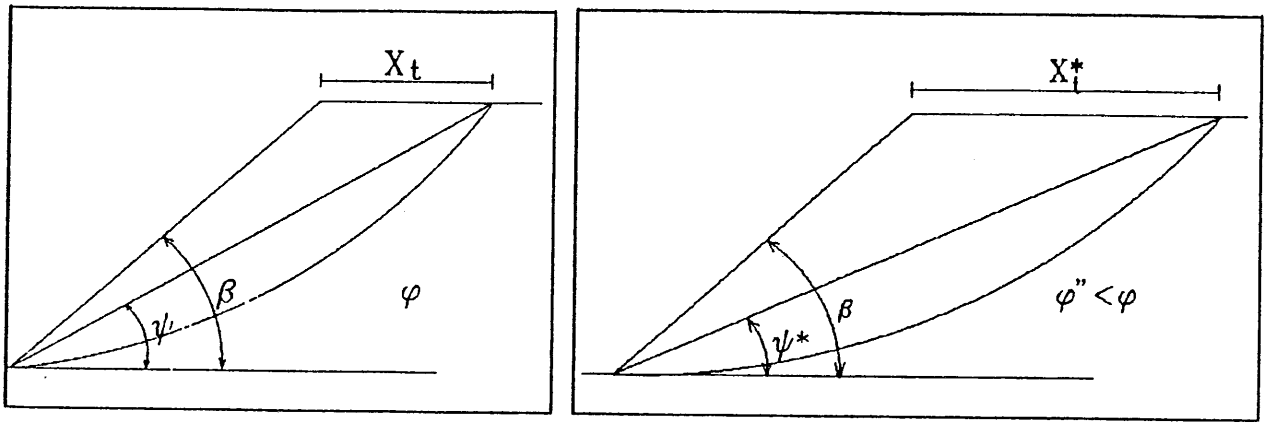

Figure 7 - Effect of Varying Friction Angle

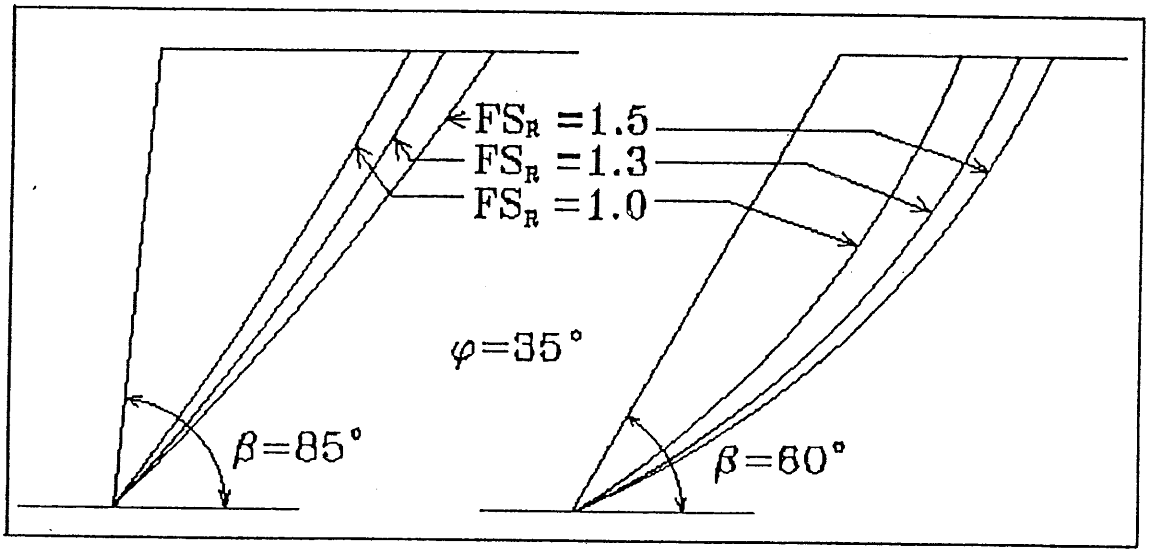

Figure 8 - Effect of Varying Slope Angle

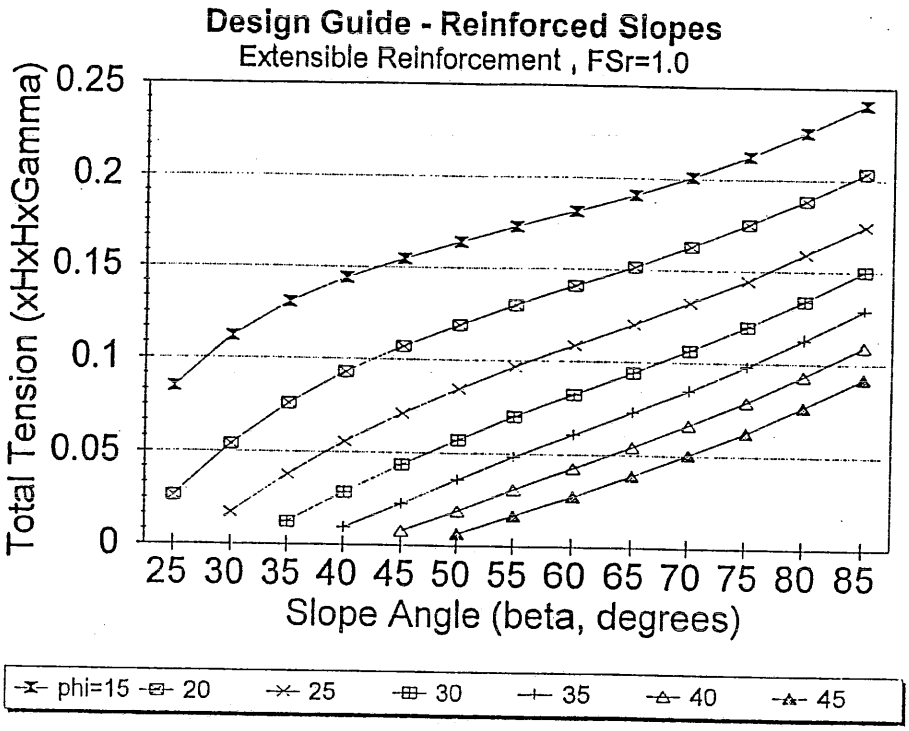

Figure 9 - Tension Required for Various Slopes, Various Friction Angles, $FS_R = 1.0$

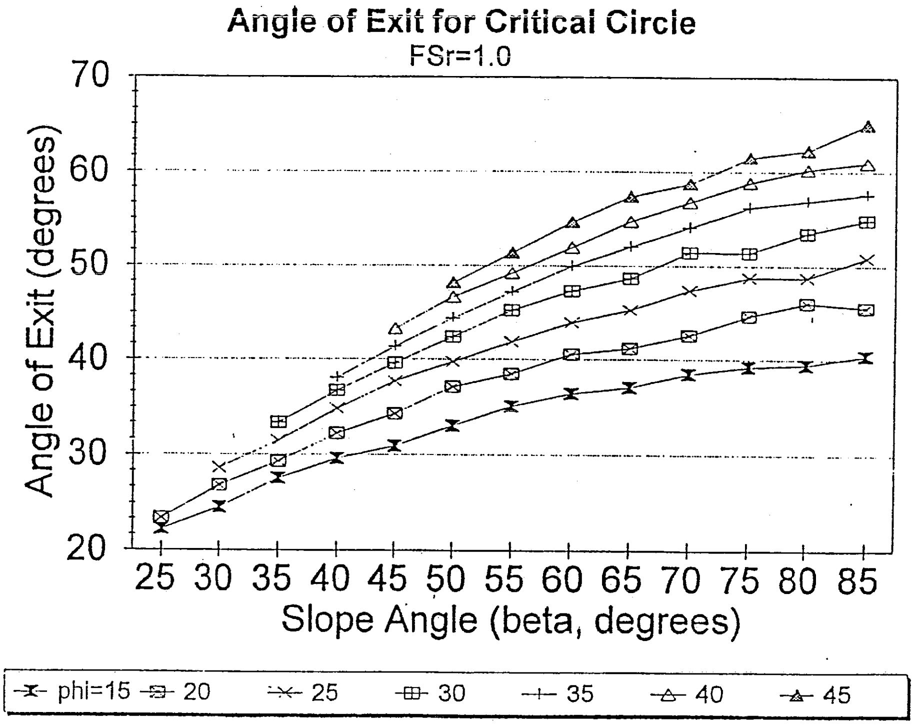

Figure 10 - Angle of Exit for Critical Circle, Various Slope and Friction Angles, $FS_R = 1.0$

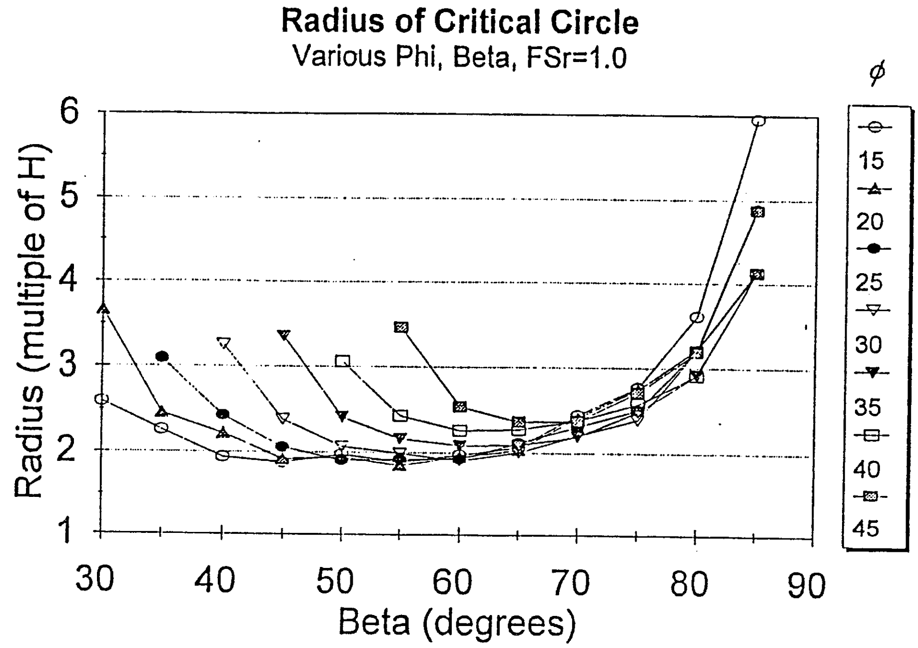

Figure 11 - Radius of Critical Circle, Various Slope and Friction Angles, FS_R = 1.0

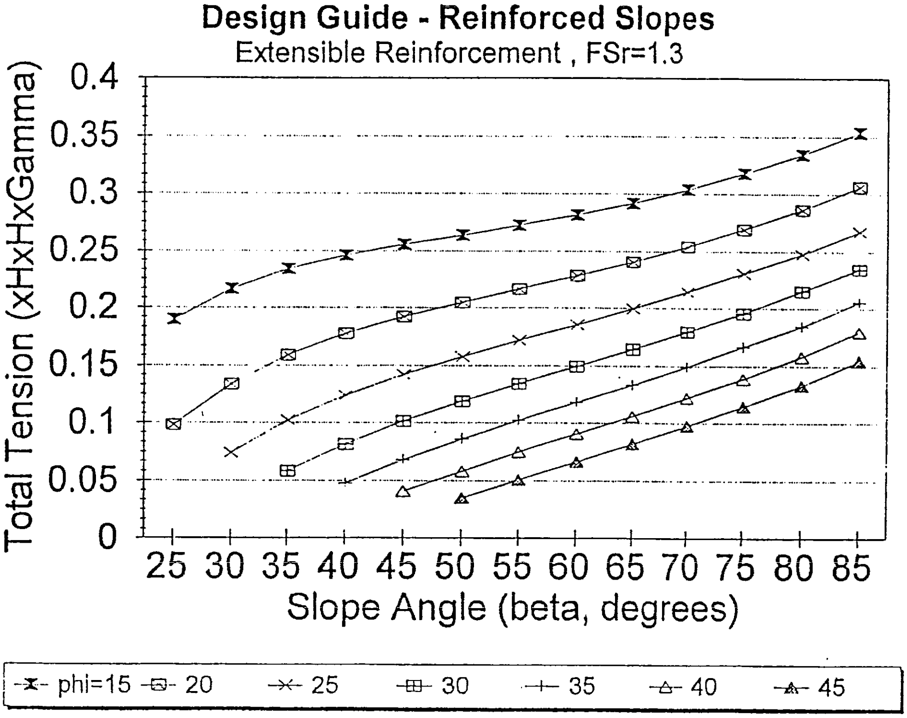

Figure 12 - Tension Required for Various Slopes, Various Friction Angles, FS_R = 1.3

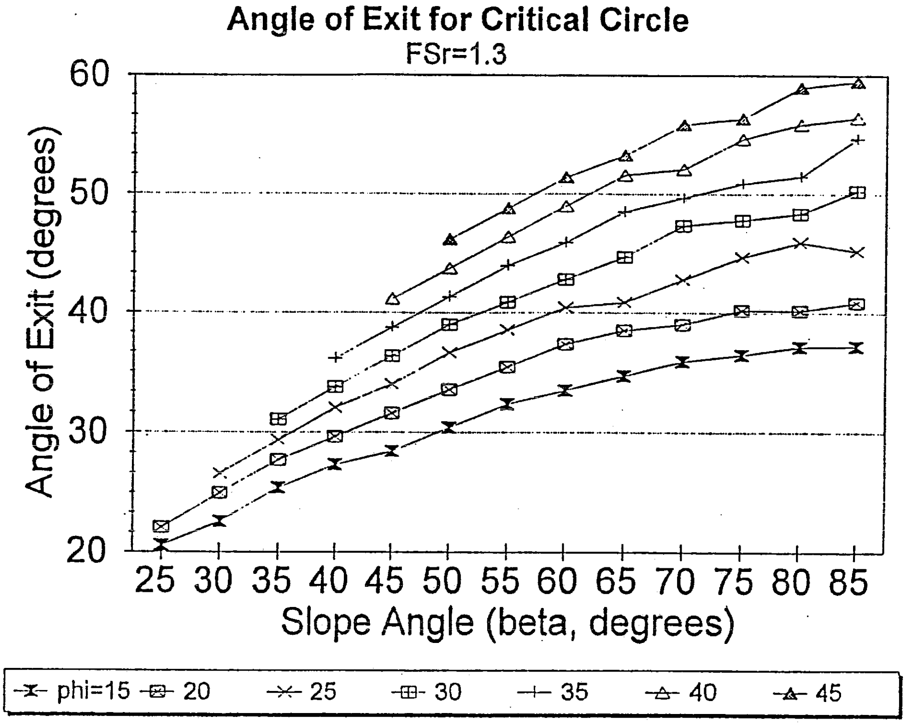

Figure 13 - Angle of Exit for Critical Circle, Various Slope and Friction Angles, FS_R = 1.3

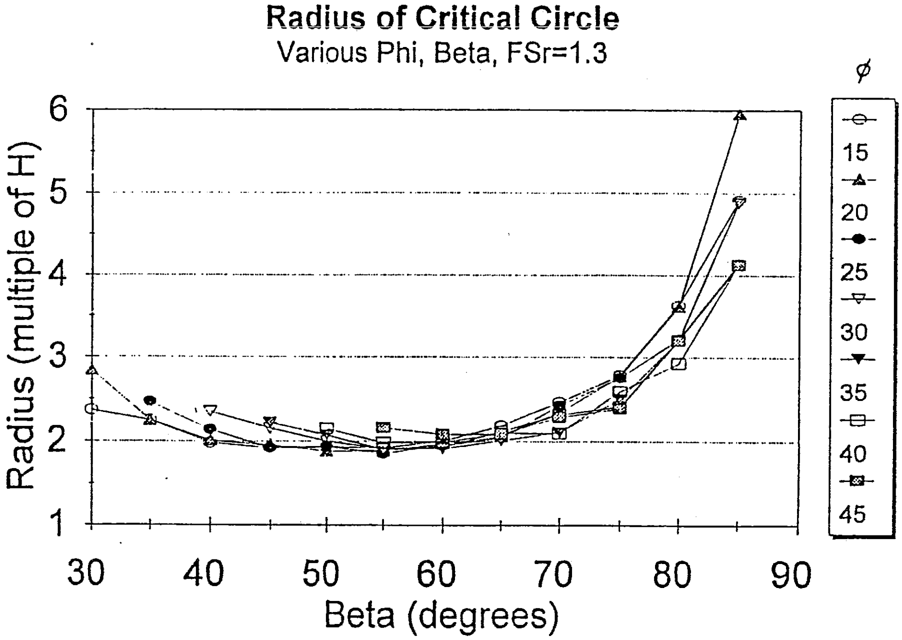

Figure 14 - Radius of Critical Circle, Various Slope and Friction Angles, FS_R = 1.3

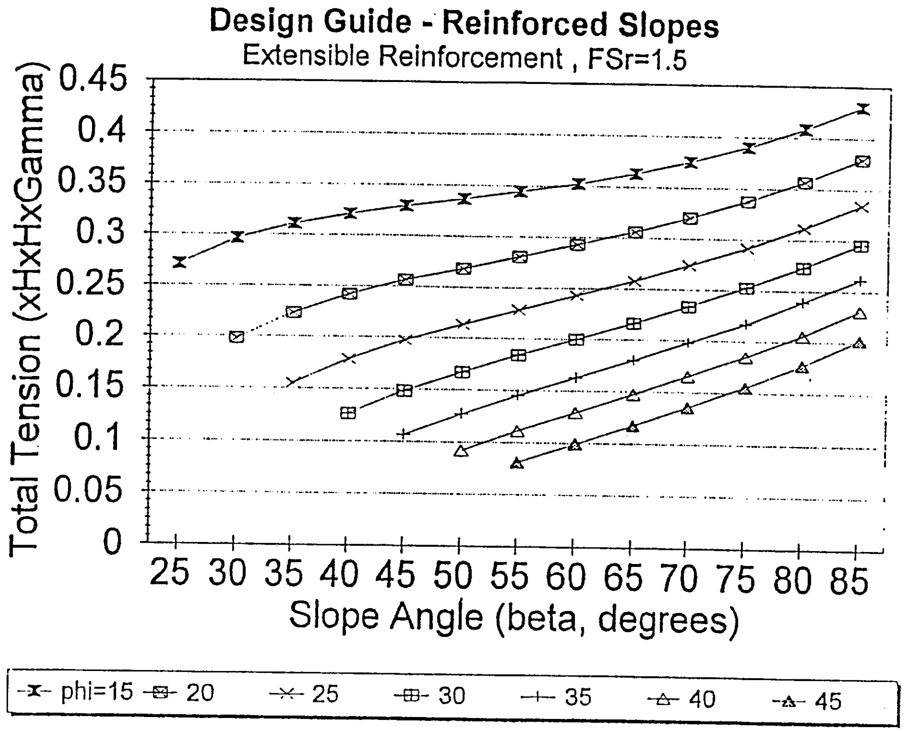

Figure 15 - Tension Required for Various Slopes, Various Friction Angles, FS_R = 1.5

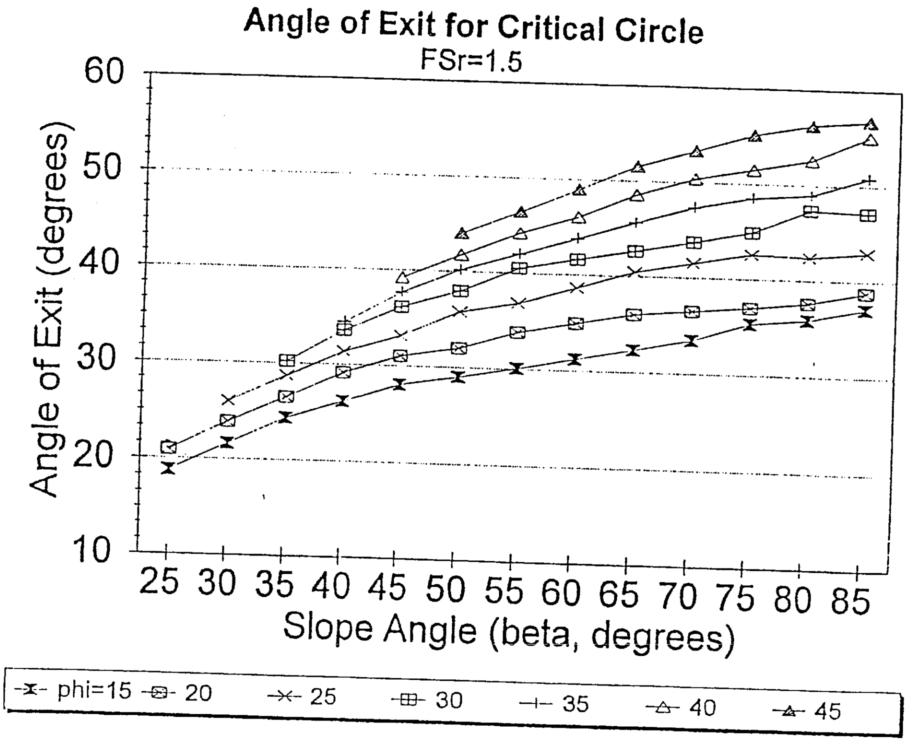

Figure 16 - Angle of Exit for Critical Circle, Various Slope and Friction Angles, FS_R = 1.5

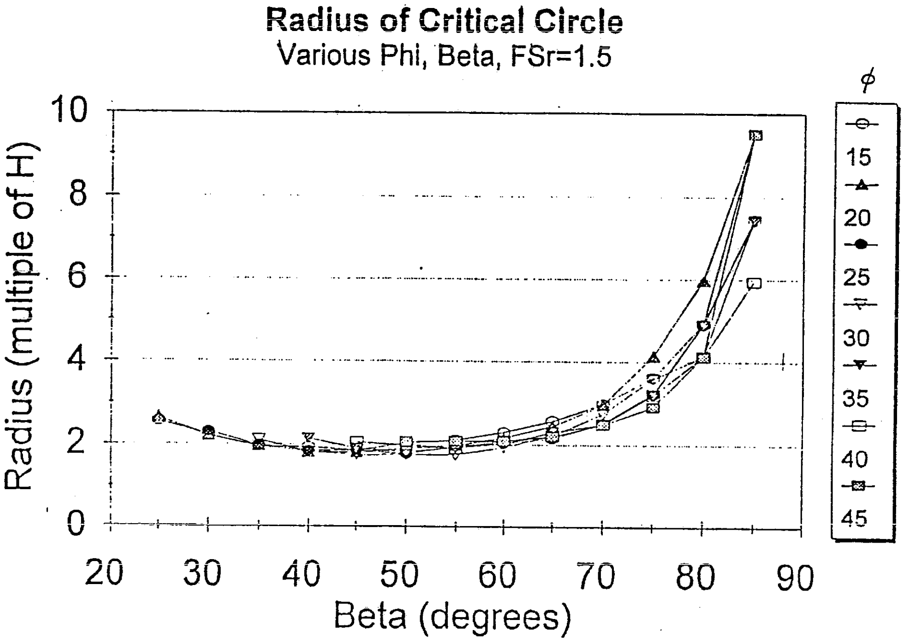

Figure 17 - Radius of Critical Circle, Various Slope and Friction Angles, $FS_R = 1.5$